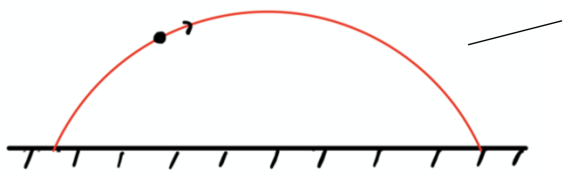
Projectile Motion Cheat Sheet

In this chapter, you will learn how to solve problems involving a particle moving only under the influence of gravity. This type of motion is known as projectile motion.

The constant acceleration (sometimes called SUVAT) formulae are of great importance in this chapter.



A typical example of projectile motion

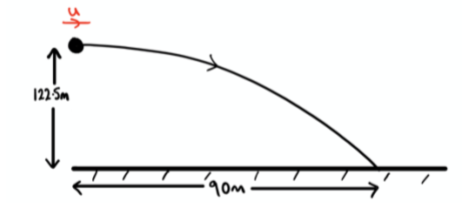
Analysing projectile motion

When analysing the motion of a projectile, it is useful to consider the horizontal and vertical motion separately.

- The horizontal motion of a projectile is modelled as having constant velocity (acceleration is 0). You can use $s = ut$.
- The vertical motion of a projectile is modelled as having constant acceleration due to gravity (acceleration $= g = 9.8$). You can use any of the constant acceleration formulae.
- When projected horizontally, the initial vertical velocity is 0 m/s.

The reason that there is no acceleration in the horizontal direction is since gravity acts vertically downwards, there are no forces acting on the particle in the horizontal direction and so there is no acceleration in this direction.

Example 1: A particle projected horizontally with speed $u\ ms^{-1}$ from a point 122.5m above a horizontal plane. The particle hits the plane at a point which is at a horizontal distance of 90m away from the starting point. Find the initial speed of the particle.



Resolving vertically:

$$\left.\begin{aligned} s &= 122.5 \\ u &= 0 \\ v &= \\ a &= g \\ t &= t \end{aligned}\right\} \quad \begin{aligned} &\text{Using } s = ut + \tfrac{1}{2}at^2: \\ &122.5 = 0 + \tfrac{1}{2}gt^2 \\ &\therefore t = \sqrt{\tfrac{2(122.5)}{g}} = 5 \end{aligned}$$

Resolving horizontally:

$$s = ut:\ 90 = ut \quad \Rightarrow \quad u = \frac{90}{t} = \frac{90}{5} = 18\ ms^{-1}$$

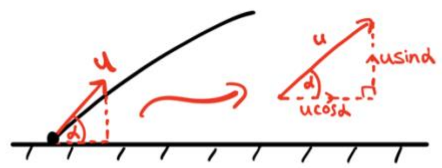
Projection at an angle

You need to be able to solve problems where a particle is projected at any angle, by considering the horizontal and vertical components of the initial velocity.

If a particle is projected with speed U at an angle α, then

- The horizontal component of the initial velocity is $Ucos\alpha$, and the vertical component is $Usin\alpha$.

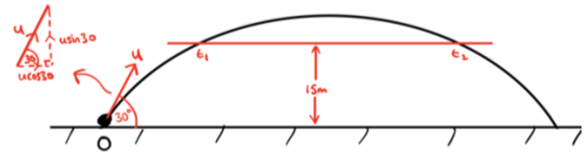
The above bullet point follows from using basic trigonometry with the right-angled triangle made by the initial velocity and the plane:



Some problems will involve finding the range of a projectile, its time of flight or the greatest height above a plane. You need to know that:

- The range of a projectile is the distance from the point of projection to the point where it strikes the horizontal plane.
- The time of flight is the total time elapsed between the initial projection and the first point of contact with the plane.
- A projectile will reach its point of greatest height when the vertical component of velocity is equal to 0.

Example 2: A particle is projected from a point O with speed $35ms^{-1}$ at an angle of elevation of 30°. The particle moves freely under gravity. Find the length of time for which the particle is 15m or more above O.



Resolving vertically to find the times when vertical distance is 15m:

$$\left.\begin{aligned} s &= 15 \\ u &= 35sin30 \\ v &= \\ a &= -g \\ t &= t \end{aligned}\right\} \quad \begin{aligned} &\text{Using } s = ut + \tfrac{1}{2}at^2: \\ &\Rightarrow 15 = 35sin30(t) - \frac{1}{2}gt^2 \\ &\Rightarrow 49t^2 - 175t + 150 = 0 \qquad \text{(simplifying)} \\ &\Rightarrow t = \tfrac{10}{7}, \tfrac{15}{7} \text{ so } t_1 = \tfrac{10}{7} \text{ and } t_2 = \tfrac{15}{7}. \qquad \text{(solving)} \end{aligned}$$

Time required $= t_2 - t_1 = \frac{5}{7}$

Projectile motion formulae

Some questions may require you to derive formulae related to projectile motion. These questions rely on the same methods we have already covered, but it is important that you are familiar with this style of question.

Example 3: A particle is projected from a point with speed U at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x, its height above the point of projection is y.

Show that $y = xtan\alpha - \frac{gx^2}{2u^2}(1 + tan^2\alpha)$

We begin by analysing the horizontal motion:

$$s = ut:\quad x = (Ucos\alpha)t \qquad [1]$$

Now looking at the vertical direction:

$$\left.\begin{aligned} s &= y \\ u &= Usin\alpha \\ v &= \\ a &= -g \\ t &= t \end{aligned}\right\} \quad \begin{aligned} &\text{Using } s = ut + \tfrac{1}{2}at^2: \\ &\Rightarrow y = Utsin\alpha - \frac{1}{2}gt^2 \qquad [2] \end{aligned}$$

We now look at the required equation: there is y in terms of x with no t anywhere. But we have two equations, y and x, both in terms of t. To form an equation involving y and x, we can make t the subject of [1] and substitute into [2]:

$$x = (Ucos\alpha)t \quad \Rightarrow \quad t = \frac{x}{Ucos\alpha}$$

Substituting into [2]:

$$\Rightarrow y = Usin\alpha\left(\frac{x}{Ucos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{Ucos\alpha}\right)^2$$

$$\Rightarrow y = x\frac{sin\alpha}{cos\alpha} - \frac{gx^2}{2U^2cos^2\alpha}$$

$$\Rightarrow y = xtan\alpha - \frac{gx^2}{2U^2}(sec^2\alpha) \qquad \text{since } \frac{1}{cos^2\alpha} = sec^2\alpha$$

since $\frac{sin\alpha}{cos\alpha} = tan\alpha$

$$\Rightarrow y = xtan\alpha - \frac{gx^2}{2u^2}(1 + tan^2\alpha) \qquad \text{since } sec^2\alpha = 1 + tan^2\alpha$$